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SOLUTIONS AND SIMPLE PROGRAMS TO PERFORM ACCURATE CALCULATIONS BASED ON RICE DISTRIBUTION

by

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ABSTRACT. The Rice distribution arises in bombing problems where it is referred to as the "offset circle probability function." It also occurs in radar as a basic relationship involving probability of false alarm (PFA), probability of detection (P_d), and signal-power-to-noise-power (S/N) ratio. Evaluation of P_d when PFA and S/N are known, or of S/N when PFA and P_d are known, is of usefulness frequently, but available means in the form of curves, approximations, and tables are not very satisfying. This report presents the solution of the integral in a form that is interesting and easily programmable. Programs in BASIC and for the Hewlett Packard 9100A desk calculator are also included.

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FOREWORD

This report derives solutions to the Rice distribution and contains simple computer programs for solving the integral for the defined dependent variable. This work was accomplished under AirTask A30-303/2161/W1174-000-01.

The report has been reviewed for technical content by J. S. Dinsmore, Jr. and R. B. Seeley.

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CONTENTS

INTRODUCTION

The Rice distribution arises in bombing problems where it is referred to as the "offset circle probability function." It also occurs in radar as a basic relationship involving probability of false alarm (PFA), probability of detection (P_d), and signal-power-to-noise-power (S/N) ratio. Evaluation of P_d when PFA and S/N are known, or of S/N when PFA and P_d are known, is of usefulness frequently, but within the writer's knowledge, available means, in the form of curves, approximations, and tables are not very satisfying. The solution of the integral in a form that is interesting and easily programmable is presented herein. Programs in BASIC and for the newly available Hewlett Packard 9100A desk calculator are also included.

DISCUSSION OF INTEGRAL VARIATIONS

The integral that concerns us appears with numerous minor variations in several references. Six of these variations are as follows:

Equation 1 is the offset circle probability function.

$$P(a,v) = \int_0^v xe^{-(a^2 + x^2)/2} I_o(ax)\, dx \qquad (1)$$

Reference 1 on page 15, Eq. 1.5 gives

$$P_d = (1/\psi_o) \int_{E_t}^{\infty} E_n \exp\left(\frac{-E_n^{\,2} - E_s^{\,2}}{2\psi_o}\right) I_o\left(\frac{E_n E_s}{\psi_o}\right) dEn \qquad (2)$$

Reference 2 on page 407, substituting Eq. 7-144 into Eq. 7-143 gives

$$P_{sb} = \int_b^{\infty} \frac{e^{-s^2}\, re^{-r^2/2N}}{N} I_o\left(rs\sqrt{2/N}\right) dr \qquad (3)$$

Reference 3 on page 170, Eq. 33.43 gives

$$D = \int_{\varepsilon_*}^{\infty} \frac{\varepsilon}{\sigma^2} e^{-[(\bar{\phi})^2 + \varepsilon^2]/2\sigma^2} I_o\left(\frac{\bar{\phi}\varepsilon}{\sigma^2}\right) d\varepsilon \qquad (4)$$

Reference 4 on page 126, Eq. 1-35 gives

$$P_{sb} = \int_{\sqrt{-\log_e P_{nb}}}^{\infty} 2\, y\, e^{-(y^2 + s^2)} I_o(2sy)\, dy \qquad (5)$$

Reference 5 on page 33, Eq. 2.28 gives the following, which is also the form used by Rice in his original work.

$$P_d = \int_{V_t}^{\infty} \frac{R}{\psi_o} \exp\left(-\frac{R^2 + A^2}{2\psi_o}\right) I_o\left(\frac{RA}{\psi_o}\right) dR \qquad (6)$$

Herein, we will start with Eq. 7 which, except for using P_d in place of P_{sb}, is the same as Eq. 3. The choice is just happenstance.

$$P_d = \int_b^{\infty} \frac{e^{-s^2}\, re^{-r^2/2N}}{N} I_o(rs\sqrt{2/N})\, dr \qquad (7)$$

Equations 1-6 are related to Eq. 7 as follows:

In Eq. 1

$$P(a,v) = 1 - P_d$$

$$x = r/\sqrt{N}, \quad dx = dr/\sqrt{N}, \quad v = b/\sqrt{N}$$

$$a^2/2 = S^2, \quad a = \sqrt{2}\, S$$

In Eq. 2

$$E_n = r, \quad dE_n = dr$$

$$E_t = b, \quad P_d = P_d$$

$$\psi_o = N, \quad E_s{}^2/2\psi_o = S^2$$

In Eq. 3

$$P_{sb} = P_d$$

In Eq. 4

$$\varepsilon = r, \quad \varepsilon_* = b$$

$$\sigma^2 = N, \quad \bar{\phi} = S\sqrt{2N}, \quad D = P_d$$

In Eq. 5

$$\sqrt{-\log_e P_{nb}} = \frac{b}{\sqrt{2N}}$$

$$y^2 = r^2/2N, \quad y = r/\sqrt{2N}, \quad dy = dr/\sqrt{2N}$$

In Eq. 6 (essentially the same as 2)

$$R = r, \quad V_T = b$$

$$\psi_o = N, \quad A^2/2\psi_o = S^2$$

In Eq. 1-7, the modified Bessel function of the first kind and zero order $I_o(Z)$ appears, and its expansion in series form, as found in handbooks or in Ref. 1-5, is given as follows:

$$I_o(Z) = \sum_{n=0}^{\infty} \frac{Z^{2n}}{2^{2n}(n!)^2} \tag{8}$$

Substituting Eq. 8 into Eq. 7 and replacing Z with rs $\sqrt{2/N}$ gives

$$P_d = \int_b^\infty \frac{e^{-s^2} re^{-r^2/2N}}{N} \sum_{n=0}^{\infty} \frac{(r\ s\ \sqrt{2/N})^{2n}}{2^{2n}(n!)^2} dr \qquad (9)$$

In Eq. 9, since we are integrating with respect to r, and summing with respect to n, we can take e^{-s^2}/N outside the integral and take $re^{-r^2/2N}$ inside the summation to give the following:

$$P_d = \frac{e^{-s^2}}{N} \int_b^\infty \sum_{n=0}^{\infty} \frac{(s\sqrt{2/N})^{2n}\ e^{-r^2/2N}\ r^{2n+1}}{2^{2n}\ (n!)^2} dr \qquad (10)$$

In Eq. 10, the s and r functions that appear inside the summation sign have also been separated.

The integration and summation in Eq. 10 can be interchanged in order and, since we are integrating with respect to r, we can write the following.

$$P_d = \frac{e^{-s^2}}{N} \sum_{n=0}^{\infty} \left[\frac{(s\sqrt{2/N})^{2n}}{(2)^{2n}(n!)^2} \int_b^\infty e^{-r^2/2N}\ r^{2n+1}\ dr \right] \qquad (11)$$

In Eq. 11 consider the integral

$$\int_b^\infty e^{-r^2/2N}\ r^{2n+1}\ dr \qquad (12)$$

Let

$$y = -r^2/2N \text{ or } r^2 = -2Ny$$

Then

$$dy = -\frac{r}{N}\,dr$$

or

$$dr = -\frac{N}{r}\,dy$$

By making these substitutions into Eq. 12 we get Eq. 13.

$$-N\int_{r=b}^{r=\infty} e^{y}(-2Ny)^{n}\,dy \tag{13}$$

Expressing Eq. 13 operationally we get

$$\left[-N\,\frac{1}{D}\,e^{y}(-2Ny)^{n}\right]_{r=b}^{r=\infty} \tag{14}$$

$$= \left[-N\,e^{y}\,\frac{1}{D+1}\,(-2Ny)^{n}\right]_{r=b}^{r=\infty} \tag{15}$$

$$= \left[-N\,e^{y}\,(1 - D + D^{2} - D^{3} + D^{4} - D^{5}----)(-2Ny)^{n}\right]_{r=b}^{r=\infty} \tag{16}$$

$$= \left[-N\,e^{y}\left\{(-2Ny)^{n} - n(-2N)(-2Ny)^{n-1} + (n)(n-1)(-2N)^{2}(-2Ny)^{n-2}---\right\}\right]_{r=b}^{r=\infty} \tag{17}$$

$$= \left[-Ne^{y} \sum_{k=0}^{n} (-1)^k \frac{n!}{(n-k)!} (-2N)^k (-2Ny)^{n-k} \right]_{r=b}^{r=\infty} \tag{18}$$

$$= \left[-Ne^{-\frac{r^2}{2N}} \sum_{k=0}^{n} (2N)^k \frac{n!}{(n-k)!} (r^2\ ^{n-k} \right]_{r=b}^{r=\infty} \tag{19}$$

$$= Ne^{-\frac{b^2}{2N}} \sum_{k=0}^{n} (2N)^k \frac{n!}{(n-k)!} (b^2)^{n-k} \tag{20}$$

Substituting Eq. 20 for the integral in Eq. 11 we get the following.

$$P_d = e^{-s^2} e^{-\frac{b^2}{2N}} \sum_{n=0}^{\infty} \left[\frac{(s\sqrt{2/N})^{2n}}{2^{2n}(n!)^2} \sum_{k=0}^{n} (2N)^k \frac{n!}{(n-k)!} (b^2)^{n-k} \right] \tag{21}$$

or

$$P_d = e^{-s^2} e^{-\frac{b^2}{2N}} \sum_{n=0}^{\infty} S^{2n} \sum_{k=0}^{n} \frac{2^n 2^k N^k (b^2)^{n-k}}{N^n (2)^{2n}\ n!(n-k)!} \tag{22}$$

or

$$P_d = \sum_{n=0}^{\infty} \frac{(S^2)^n}{e^{S^2} n!} \sum_{k=0}^{n} \frac{(b^2/2N)^{n-k}}{e^{b^2/2N}(n-k)!} \tag{23}$$

In Eq. 23, S^2 is the signal-power-to-noise-power ratio, and $e^{-b^2/2N}$ is the PFA. Using R for the signal-to-noise ratio in place of S^2 and T in place of $b^2/2N$, we get the following:

$$P_d{}' = \sum_{n=0}^{\infty} \frac{R^n}{e^R n!} \sum_{k=0}^{n} \frac{T^{n-k}}{e^T (n-k)!} \tag{24}$$

The form of Eq. 24 is noteworthy in that for each value of n, the second summation is the ratio of the sum of the first n terms of the e^T series divided by the entire e^T series, and the term between the summation signs acting as a factor is the ratio of the n^{th} term of the e^R series divided by the entire e^R series.

In programming Eq. 24, it will help to write out the development of P_d for the first few values of n.

n	k	Partial Sums
0	0	$(1/e^R)(1/e^T)$
1	0, 1	$(R/e^R)\{(1 + T)/e^T\}$
2	0, 1, 2	$\{R^2/(e^R 2!)\}\{(1 + T + T^2/2!)/e^T\}$
.	.	.
.	.	.
.	.	.
.	.	.
n	0, 1, ----n	$\{R^n/(e^R n!)\}\{(1 + T + T^2/2! + \text{----}\ T^n/n!)/e^T\}$

We are to find P_d with the following given:

$$(S/N) \text{ in db} = 10 \log_{10} R$$

$$PFA = 1/e^T$$

We define and use the following calculations where n takes integral values starting with zero.

$$F1 = R^n/(e^R n!)$$

$$F0 = T^n/(e^T n!)$$

$$F2 = (1 + T + T^2/2! + \text{----}\ T^n/n!)/e^T$$

$$\delta = (F1)(F2)$$

Each of the partial sums above is made up of two factors: the factor involving R and the factor involving T. We call the R factor "F1" and the T factor "F2." Succeeding F1 factors are obtained by multiplying the preceding F1 factor by R/n. Succeeding F2 factors are obtained by adding a new term to the preceding F2 factor; we call this additive increment "F0." Succeeding F0 increments are obtained by multiplying the preceding F0 increment by T/n.

In the program, we call the partial sums "δ" because they are additive increments to the previous sum to get the next approximation to P_d. These δ terms can be very small for initial values of n and therefore we cannot use a simple comparison of a newly calculated δ with a small number as a test for stopping the calculation. We compare each δ with the previous δ and continue as long as δs are increasing. When the δs start decreasing we terminate when we reach a $\delta \leq 10^{-11}$ value, which goes beyond needed accuracy and can be altered if desired, although the required number of executions will not be greatly reduced.

A program for Eq. 24 in BASIC is as follows:

```
10    READ PO, R4    [where PO=PFA, R4=(S/N) in db; both given]
20    LET  T= -LOG(PO)  [where LOG is to base e]
30    LET  R=10↑(.1*R4)
40    LET  P4=0   [where P4=Pd; to be calculated]
50    LET  D1=0   [where D1=δ; see D2 below]
60    LET  M=0    [where M=n]
80    LET  F1=EXP(-R)
100   LET  F2=0
120   LET  F0=P0
160   LET  F2=F2 + F0
180   LET  D2=F1*F2   [where D2=δ, D1 and D2 are succeeding values of δ]
200   LET  P4=P4 + D2
220   LET  M=M + 1
240   LET  F1=(R/M)*F1
260   LET  F0=(T/M)*F0
280   IF   D2>D1 THEN 340
300   IF   D2>1E-11 THEN 340
320   GO TO 380
340   LET  D1=D2
360   GO TO 160
380   PRINT "PFA= "; P0, "S/N= "; R4, "PD= "; P4
400   GO to 10
```

500 DATA ---- [Enter PFA, P_d in pairs]

.
.
.
.

999 END

Finding (S/N) = R when PFA = $1/e^T$ and P_d are given can be done as follows:

Let

$$f(R) = \sum_{n=0}^{\infty} \frac{R^n}{e^R n!} \sum_{R=0}^{n} \frac{T^{n-k}}{e^T[(n-k)!]} - P_d \tag{25}$$

$$= P_d(R) - P_d$$

We wish to find the value of R that makes f(R) = 0 in Eq. 25. Taking P_d and T as constants we get:

$$\frac{df(R)}{dR} = \sum_{n=0}^{\infty} \frac{n\,R^n}{e^R Rn!} \sum_{k=0}^{n} \frac{T^{n-k}}{e^T[(n-k)!]} - P_d(R) \tag{26}$$

$$= 1/e^R\ 1/e^T[0+1(T+1)+R(T^2/2!+T/1!+1)$$

$$+ R^2/2!(T^3/3!+T^2/2!+T/1!+1) \text{ ----}]$$

$$- 1/e^R\ 1/e^T[1+R(T+1)+R^2/2!(T^2/2!+T/1!+1)$$

$$+ R^3/3!(T^3/3!+T^2/2!+T/1!+1) \text{ ----}]$$

or

$$\frac{df(R)}{dR} = 1/e^R\ 1/e^T \left[T + \frac{R}{1!}\frac{T^2}{2!} + \frac{R^2}{2!}\frac{T^3}{3!} + \frac{R^3}{3!}\frac{T^4}{4!} \text{ ----}\right] \tag{27}$$

If we make an initial estimate of R = R_1, we can use the Newton-Raphson iterative procedure (Eq. 28) to get succeeding approximations.

$$R_2 = R_1 - f(R_1)/f^1(R_1) \tag{28}$$

Equation 24 programmed for the H.P. 9100A follows:

Step	Key	Code	Display			Storage					
			x	*y*	*z*	*f*	*e*	*d*	*c*	*b*	*a*
0 0	CLR	20									
0 1	STOP	41	PFA	(S/N) in db	Enter (S/N)	in db	and PFA				
0 2	x→()	23									
0 3	e	12									
0 4	x→()	23									
0 5	d	17									
0 6	ln x	65									
0 7	Chg. sign	32									
0 8	x→()	23									
0 9	b	14									
0 *a*	1	01									
0 *b*	0	00									
0 *c*	÷	35									
0 *d*	ln x	65									
1 0	X	36									
1 1	+	25									
1 2	e^x	74									
1 3	x→()	23									
1 4	a	13									
1 5	Chg. sign	32									
1 6	e^x	74									
1 7	x→()	23									
1 8	c	16									
1 9	↑	27									
1 *a*	e	12									
1 *b*	X	36									
1 *c*	y→()	40									
1 *d*	9	11									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
2 0	1	01									
2 1	x→()	23									
2 2	f	15									
2 3	↓	25									
2 4	Roll ↑	22									
2 5	+	33									
2 6	Pause	57									
2 7	Con- tinue	47									
2 8	Con- tinue	47									
2 9	↑	27									
2 a	a	13									
2 b	x⇄y	30									
2 c	f	15									
2 d	÷	35									
3 0	c	16									
3 1	X	36									
3 2	y→()	40									
3 3	c	16									
3 4	b	14									
3 5	x⇄y	30									
3 6	f	15									
3 7	÷	35									
3 8	d	17									
3 9	X	36									
3 a	y→()	40									
3 b	d	17									
3 c	1	01									
3 d	Acc +	60									

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
40	c	16									
41	x⇄y	30									
42	e	13									
43	X	36									
44	↓	25									
45	↑	27									
46	y⇄()	24									
47	9	11									
48	x⇄y	30									
49	IF x<y	52									
4a	2	02									
4b	3	03									
4c	Enter Exp.	26									
4d	Chg. sign	32									
50	1	01									
51	1	01									
52	IF x<y	52									
53	2	02									
54	3	03									
55	↓	25									
56	+	33									
57	a	13									
58	LOG x	75									
59	↑	27									
5a	1	01									
5b	0	00									
5c	X	36									
5d	b	14									

Step		Key	Code	Display			Storage					
				x	y	z	f	e	d	c	b	a
6	0	Chg. sign	32									
	1	e^x	74									
	2	END	46	PFA	(S/N) in db	P_d	Final display					
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

The function f(R) plotted as the ordinate with R as the abscissa lies between $f(R) = PFA - P_d$ and an upper asymptote $f(R) = 1 - P_d$; at R = 0, $f(R) = PFA - P_d$ with a slope T/e^T (where $1/e^T = PFA$) and rises, changing from concave upward to concave downward, toward its upper asymptote. Because of this shape, the repetitive use of Eq. 28 will converge only when the initial R_1 has suitable values. Two programs, one in BASIC and one for the H.P. 9100A, are presented below. In the BASIC program, a rough initial estimate of R is generated. In the H.P. 9100A program, a value for R_1 must be entered and changed if necessary to secure convergence. The BASIC program, as written, finds S/N in db to the nearest 0.01 db. The H.P. 9100A program presents the result following each iteration showing R_1 and R_2 for comparison; pressing "CONTINUE" proceeds to the next approximation so that the full accuracy of the calculator is attainable. When the two values are adequately alike, setting the flag and pressing "CONTINUE" gives a display of S/N in decibels. A separate program for the H.P. 9100A is given also to generate an initial value for R_1. The BASIC program, as given, has been run on the GE Mark I time-sharing system for a wide range of PFA and P_d values, but it operates for only a limited range of values on the GE Mark II time-sharing system on which it produces underflows that sometimes lead to divisions by zero.

CALCULATION OF S/N WHEN PFA AND P_d ARE GIVEN

All alpha registers and register 9 are used for storage. If new estimate R_2 diverges, insert an intermediate estimate in the y register before pressing "CONTINUE" at program step 71 (not necessary to preserve x and z displays). If R_2 is negative, zero can be used as a new R_2.

Step		Key	Code	Display x	 y	EST. R z	$n+P_d$ f	$P_d(R)$ e	$f^1(R)$ d	rageF c	FO b	T a
0	0	CLEAR	20									
0	1	STOP	41	$1 + P_d$	PFA	R_1	Enter estimate of numerical value of $S/N=R_1$; probability of false alarm = PFA; one plus probability of detection $(P_d) = 1 + P_d$					
0	2	ACC +	60									
0	3	e	12									
0	4	ln x	65									
0	5	Chg. sign	32									
0	6	x→()	23									
0	7	a	13									
0	8	÷	25									
0	9	x⇄y	30									
0	a	Chg. sign	32									
0	b	e^x	74									
0	c	x→()	23									
0	d	c	16									
1	0	y→()	40									
1	1	b	14									
1	2	X	36									
1	3	y→()	40									
1	4	9	11									
1	5	y→()	40									
1	6	e	12									
1	7	0	00									
1	8	x→()	23									
1	9	d	17									
1	a	a	13									
1	b	x⇄y	30									
1	c	f	15									
1	d	INT x	64									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
2 0	÷	35									
2 1	b	14									
2 2	X	36									
2 3	y→()	40									
2 4	b	14									
2 5	c	16									
2 6	X	36									
2 7	d	17									
2 8	x⇄y	30									
2 9	+	33									
2 a	y→()	40									
2 b	d	17									
2 c	ROLL ↓	31									
2 d	f	15									
3 0	INT x	64									
3 1	ln x	65									
3 2	Chg. sign	32									
3 3	e^x	74									
3 4	x⇄y	30									
3 5	X	36									
3 6	ROLL ↑	22									
3 7	y⇄()	24									
3 8	9	11									
3 9	x⇄y	30									
3 a	+	33									
3 b	ROLL DOWN	31									
3 c	X	36									
3 d	÷	25									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
4 0	↑	27									
4 1	y⇄()	24									
4 2	9	11									
4 3	ROLL ↑	22									
4 4	IF x<y	52									
4 5	5	05									
4 6	1	01									
4 7	ENTER EXP	26									
4 8	Chg. sign	32									
4 9	1	01									
4 *a*	1	01									
4 *b*	IF x<y	52									
4 *c*	5	05									
4 *d*	1	01									
5 0	SET FLAG	54									
5 1	0	00									
5 2	ACC +	60									
5 3	↓	25									
5 4	f	15									
5 5	INT x	64									
5 6	÷	35									
5 7	c	16									
5 8	X	36									
5 9	y→()	40									
5 *a*	c	16									
5 *b*	0	00									
5 *c*	x⇄y	30									
5 *d*	1	01									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6 0	ACC +	60									
6 1	IF FLAG	43									
6 2	6	06									
6 3	7	07									
6 4	GO TO	44									
6 5	1	01									
6 6	a	13									
6 7	RCL	61									
6 8	-	34									
6 9	INT x	64									
6 a	+	33									
6 b	d	17									
6 c	÷	35									
6 d	↓	25									
7 0	-	34									
7 1	STOP	41		R_2	R_1	New estimate of (S/N) appears					
7 2	f	15				in y register with prior esti-					
7 3	↑	27				mate in the z register. Press					
7 4	INT x	64				"CONTINUE" to go to next esti-					
7 5	-	34				mate. When R_1 and R_2 are ade-					
7 6	1	01				quately alike, manually set flag					
7 7	+	33				and press "CONTINUE" to convert					
7 8	y→()	40				to decibels: see last program					
7 9	f	15				line.					
7 a	↓	25									
7 b	a	13									
7 c	Chg sign	32									
7 d	e^x	74									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
8	0	IF FLAG	43									
8	1	8	10									
8	2	6	06									
8	3	GO TO () ()	44									
8	4	0	00									
8	5	9	11									
8	6	ROLL ↓	22									
8	7	LOG x	75									
8	8	ROLL ↓	22									
8	9	1	01									
8	a	0	00									
8	b	X	36									
8	c	f	15									
8	d	END	46	1 + P_d	(S/N) in db	PFA	Final S/N in decibels displayed					
	0						in y register, 1 + P_d in x, and					
	1						PFA in z.					
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

APPROXIMATION TO S/N WHEN PROBABILITY OF FALSE ALARM AND PROBABILITY OF DETECTION ARE GIVEN

This is intended only for use as a first estimate of R in Eq. 28.
APPROXIMATE $R = \sqrt{T}\,(\sqrt{T} + 2\sqrt{2}(2P_d - 1)) + 8\,P_d(P_d - 1) + 1.5$

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0 0	CLEAR	20									
0 1	STOP	41	PFA	P_d	Enter PFA and P_d						
0 2	ln x	65									
0 3	Chg. sign	32									
0 4	$\sqrt{x}$	76									
0 5	ACC +	60									
0 6	2	02									
0 7	X	36									
0 8	↑	27									
0 9	$\sqrt{x}$	76									
0 a	X	36									
0 b	1	01									
0 c	ROLL UP	22									
0 d	x⇄y	30									
1 0	-	34									
1 1	↑	25									
1 2	X	36									
1 3	f	15									
1 4	+	33									
1 5	X	36									
1 6	1	01									
1 7	.	21									
1 8	5	05									
1 9	+	33									
1 a	e	12									
1 b	↑	27									
1 c	1	01									
1 d	-	34									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
2	0	8	10									
2	1	X	36									
2	2	e	12									
2	3	X	36									
2	4	↓	25									
2	5	+	33									
2	6	END	46		ESTIMATED R		Read estimated R in y display.					
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

A program in BASIC for Eq. 28 is as follows:

```
10    PRINT "PFA," "PD," "(S/N), DB"
15    FRINT
20    READ P0, F4
25    IF P0 = P4 THEN 365
30    IF P0 > P4 THEN 378
35    LET T = -LOG(P0)
40    LET R1 = T + SQR (8*T)*(2 P4 -1) + 8* P4*(P4 - 1) + 1.5
49    LET R2 = R1
50    IF R1 > 0 THEN 70
60    LET R1 = (P4/P0 - 1)/T
70    LET N = 1
75    LET F = 0
80    LET F0 = P0
90    LET F1 = EXP(-R1)
100   LET S = 0
110   LET D1 = 0
120   LET D = F1*P0
130   LET P = D
140   LET F0 = (T/N)*F0
150   LET S0 = F1*F0
160   LET S = S + S0
170   LET D = (R1/N)*(S0 + D)
180   IF D1 < D THEN 210
190   IF 1E-11 < D THEN 210
200   LET F = 1
210   LET D1 = D
220   LET P = P + D
230   LET F1 = (R1/N)*F1
240   LET N = N + 1
250   IF F = 1 THEN 265
260   GO TO 140
```

```
265    LET RO = (P4 -P)/S
280    IF ABS(RO) < 10 THEN 300
290    LET RO = 10*(RO/ABS(RO))
300    LET R = R1 + RO
310    IF R > 0 THEN 330
320    LET R1 = R1/2
325    GO TO 70
330    IF ABS (LOG(R/R1)) < = .00115 THEN 360
340    LET R1 = R
350    GO TO 70
360    PRINT PO, P4, INT(1000 * (LOG(R)/LOG(10)) + .5)/100
361    PRINT R2, R
362    GO TO 15
365    PRINT
370    PRINT PO, P4, "NO SIGNAL"
375    GO TO 15
378    PRINT
380    PRINT PO, P4, "PD < PFA"
385    GO TO 15
500    DATA PFA, Pd, ---- PFA, Pd, ETC.
999    END
```

CONCLUSIONS

These programs have proven useful in evaluating radar data acquired using pulse height analysis.

REFERENCES

1. Barton, David K. *Radar System Analysis*. Clifford Heights, N.J., Prentice-Hall, 1964.

2. Schwartz, Mischa. *Information Transmission, Modulation, and Noise*. New York, McGraw-Hill, 1959.

3. Wainstein L. A., and V. D. Zubakov. *Extraction of Signals from Noise*. Clifford Heights, N. J., Prentice-Hall, 1962.

4. Berkowitz, Raymond S. *Modern Radar: Analysis, Evaluation, and System Design*. New York, Wiley, 1965.

5. Skolnik, Merrill I. *Introduction to Radar Systems*. New York, McGraw-Hill, 1962.

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13. ABSTRACT

The Rice distribution arises in bombing problems where it is referred to as the "offset circle probability function." It also occurs in radar as a basic relationship involving probability of false alarm (PFA), probability of detection ($P_{\hat{d}}$), and signal-power-to-noise-power (S/N) ratio. Evaluation of $P_{\hat{d}}$ when PFA and S/N are known, or of S/N when PFA and $P_{\hat{d}}$ are known, is of usefulness frequently, but available means in the form of curves, approximations, and tables are not very satisfying. This report presents the solution of the integral in a form that is interesting and easily programmable. Programs in BASIC and for the Hewlett Packard 9100A desk calculator are also included.

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Probability of False Alarm						
Signal-to-Noise Ratio						
Computer Programs						